Multi-objective Iterative Learning Control using Convex Optimization

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Abstract

This paper presents a multi-objective iterative learning control (ILC) design approach that realizes an optimal trade-off between robust convergence, converged tracking performance, convergence speed, and input constraints. Linear time-invariant single-input single-output systems which are represented by both parametric and nonparametric models are considered. The noncausal filter $Q(\mathrm{q})$ and learning function $L(\mathrm{q})$ are simultaneously optimized by solving a convex optimization problem. The proposed method is applied to a non-minimal phase system and compared with a model-inversion based ILC design. Using the developed ILC design the underlying trade-off between tracking performance and convergence speed is thoroughly/quantitatively analyzed.

Keywords: iterative learning control, $\mathcal{H}_\infty$ control, multi-objective control, convex optimization

1. Introduction

Iterative learning control (ILC) is widely used in control applications to improve performance of repetitive processes [1, 2]. The key idea of ILC is to update the control signal iteratively based on measured data from previous trials, such that the output converges to the given reference trajectory. The purpose of this paper is to present a multi-objective iterative learning control (ILC) design. More specifically, this paper considers ILC applied to linear time-invariant (LTI), single-input single-output (SISO) systems. The setup is a standard ILC type [1]:

$$u_{j+1}(k) = Q(\mathrm{q})\left[u_j(k) + L(\mathrm{q})e_j(k+1)\right], \tag{1}$$

where $u_j(k)$ is the ILC input signal and $e_j(k)$ is the error signal between the reference trajectory and the output signal. The subscript j denotes the trial number. $Q(\mathrm{q})$ and $L(\mathrm{q})$ are known in ILC literature as the Q-filter and learning function, respectively. The choice of $Q(\mathrm{q})$ and $L(\mathrm{q})$ is the main issue in the design of an ILC algorithm.

Most ILC algorithms in the literature rely on a two-step problem formulation and the design procedures are usually heuristic. The design problem of $L(\mathrm{q})$ is usually formulated first. Various choices of $L(\mathrm{q})$ have been discussed such as P-type [3], PD-type [4], model-inversion [5–7] and phase-lead [8–10]. The first three approaches aim to find a learning function that is closest to the inverse of the system dynamics. These methods are sensitive to model uncertainties, and have difficulties dealing with non-minimum phase systems [8]. The phase-lead type ILC is based on tuning a learning gain and a linear phase-lead variable. Even though some guidelines have been provided to find the optimal variables, the tuning process is typically trial-and-error. The system is required to be reset whenever the parameters are adjusted. After $L(\mathrm{q})$ is found, $Q(\mathrm{q})$ is commonly designed as a low-pass filter. The filter characteristics (i.e. filter type, cut-off frequency and order) are selected by the designer such that robustness and high tracking performance are obtained. On the other hand, [11, 12] consider $\mathcal{H}_\infty$-based ILC methods to design the learning function for the given Q-filter. The solutions are however limited to only causal functions. Finally, [13, 14] proposed

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norm optimal ILC relying on tuning three weighting matrices. The computational cost of norm optimal ILC is however high, especially if the trial duration is long.

Generally, the described ILC methods suffer from two disadvantages. First, selection of the Q-filter and learning function are not always optimal. This is because of the nature of the two-step and heuristic techniques. Second, multiple ILC objectives, namely robust convergence, convergence speed, converged tracking performance and input constraint, are hard to consider explicitly in the design. For example, while model-inversion ILC type often yields fast convergence speed, the converged tracking error might not be satisfactory. The question is whether a smaller asymptotic tracking error can be obtained with (possibly) the compromise of lower convergence speed? This trade-off analysis has not been investigated extensively in the ILC literature.

To deal with these disadvantages, this paper proposes a one-step optimization based ILC design to solve for noncausal Q-filter and learning function simultaneously. Furthermore, multiple ILC objectives are incorporated into our algorithm. We reformulate the problem as a convex problem, guaranteeing an efficient and reliable computation of the global optimum and allowing straightforward computation of trade-off curves between different performance indices. These trade-off curves aid the control engineers in selecting their desired controller taking into account different objectives. Additionally, the proposed design can deal with both parametric and nonparametric models. For example, frequency response measurements of the system [15] can be used directly, and hence the identification of a parametric model based on this system data is not required.

The proposed ILC approach is developed in the frequency domain. Notice that the ILC analysis and design generally rely on two system representations: frequency domain and lifted-system representation. The lifted-system representation accounts for the finite trial length of the trial. However, it has a major disadvantage in computational cost since the size of the matrices depend on the number of samples N, and the cost of the optimization solvers is approximate $\mathcal{O}(N^6)$ [16]. For applications with large trial lengths, the computation of the learning matrices may take too much time, and the memory requirement may be excessive. In [17], we have also shown that the convergence condition using lifted systems could be converted into a linear matrix inequality, yet can be solved for only short trial lengths. On the other hand, the frequency domain ILC assumes an infinite trial length, and hence is an approximation of the ILC system. The advantage is that the approach requires considerably less computations and is practically more useful. These reasons motivate us to investigate the frequency domain ILC.

The remainder of this paper is organized as follows. Section II provides the background on ILC and discusses different objectives that can be considered in ILC design. Section III formulates the developed ILC design approach. Simulation results are given in Section IV, and Section V concludes this paper.

2. Problem Formulation

2.1. System formulation

The ILC design is considered in discrete time, where the discrete time instants are labeled by $k = 0, 1, \ldots$ and q denotes the forward time shift operator. Each trial comprises N time samples and prior to each trial the plant is returned to the same initial conditions. We assume that the linear system is subject to unstructured multiplicative uncertainty. That is, the method accounts for a set of uncertain systems $P_\Delta(\mathrm{q})$ of the following form:

$$P_\Delta(\mathrm{q}) = \hat{P}(\mathrm{q})(1 + \Delta(\mathrm{q})W(\mathrm{q}))\,, \quad \Delta(\mathrm{q}) \in \mathcal{B}_\Delta\,, \tag{2a}$$

with

$$\mathcal{B}_\Delta = \{\Delta(\mathrm{q}) = \text{stable, LTI system} : \|\Delta(\mathrm{q})\|_\infty \leq 1\}\;, \tag{2b}$$

where $\|.\|_\infty$ is the $\mathcal{H}_\infty$ norm. $\hat{P}(\mathrm{q})$ is the nominal plant model. And the uncertainty weighting function $W(\mathrm{q})$ determines the size of the uncertainty which is assumed to be known.

The Q-filter and the learning function in the ILC algorithm (1) are represented as their impulse responses:

$$Q(\mathrm{q}) = ... + q_{-2}\mathrm{q}^{-2} + q_{-1}\mathrm{q}^{-1} + q_0 + q_1\mathrm{q}^1 + q_2\mathrm{q}^2 + ... \tag{3}$$

and

$$L(\mathrm{q}) = ... + l_{-2}\mathrm{q}^{-2} + l_{-1}\mathrm{q}^{-1} + l_0 + l_1\mathrm{q}^1 + l_2\mathrm{q}^2 + ... \tag{4}$$

These representations allow noncausal filtering.

2.2. Robust convergence and convergence speed

We consider the robust convergence analysis in the frequency domain. The frequency response representation is obtained by replacing q with $e^{j\omega}$ for $\omega \in [-\pi, \pi]$ in (1). Accordingly, the ILC system (1) achieves robust convergence if [1]:

$$\sup_{\Delta\in\mathcal{B}_\Delta} |Q(\mathrm{q})[1 - L(\mathrm{q})P_\Delta(\mathrm{q})]| = \gamma^* < 1, \quad \forall P_\Delta(\mathrm{q}),\ \forall \mathrm{q} \in \mathcal{D}, \tag{5}$$

with

$$\mathcal{D} = \left\{\mathrm{q} = e^{j\omega} \ \mid\ \omega \in [-\pi, \pi]\right\}. \tag{6}$$

This condition also guarantees monotonic convergence with noncausal $Q(\mathrm{q})$ and $L(\mathrm{q})$ for sufficiently large trial lengths [18]. In addition, the smaller γ^*, the faster $u_j(k)$ converges to the fixed point $u_\infty(k)$ since $u_\infty(k) - u_j(k) = Q(\mathrm{q})[1 - L(\mathrm{q})P_\Delta(\mathrm{q})](u_\infty(k) - u_j(k))$. This will result in higher convergence speed and hence more economical and desirable.

2.3. Robust performance

The tracking performance of an ILC system is based on the asymptotic value of the error signal in the trial domain. Robust performance ILC requires the tracking performance specifications to be met for all plants in the uncertainty set. If the ILC system (1) is robust convergent, the asymptotic tracking error is given by:

$$e_\infty(k) = \frac{1 - Q(\mathrm{q})}{1 - Q(\mathrm{q})[1 - L(\mathrm{q})P_\Delta(\mathrm{q})]}\, y_\mathrm{d}(k), \tag{7}$$

where $y_\mathrm{d}(k)$ is the reference signal. Accordingly, the robust performance condition is defined as

$$\left| W_\mathrm{p}(\mathrm{q}) \frac{1 - Q(\mathrm{q})}{1 - Q(\mathrm{q})[1 - L(\mathrm{q})P_\Delta(\mathrm{q})]} \right| < 1,\ \ \forall P_\Delta(\mathrm{q}), \forall \mathrm{q} \in \mathcal{D}, \tag{8}$$

where $W_\mathrm{p}(\mathrm{q})$ is the performance weight selected by the designer. In other words, robust performance is satisfied if the worst-case weighted performance function at each frequency is less than 1. The nominal performance condition is identical to (8) except model uncertainty is ignored.

The ILC design usually accounts for robust convergence (5) and converged performance (7). If the system model is perfect, it is trivial to choose $L(\mathrm{q})$ as an inverse of the model and $Q(\mathrm{q}) = 1$ so that the tracking error converges to zero in one trial. This does not happen in real cases since models are never perfect and can be non-minimum phase.

2.4. Input constraint

The input constraint condition is defined based on the converged input signal in the trial domain:

$$u_\infty(k) = \frac{Q(\mathrm{q})L(\mathrm{q})}{1 - Q(\mathrm{q})[1 - L(\mathrm{q})P_\Delta(\mathrm{q})]}\, y_\mathrm{d}(k). \tag{9}$$

As a result, the condition is given similar to the robust performance condition as follows,

$$\left| W_\mathrm{u}(\mathrm{q}) \frac{Q(\mathrm{q})L(\mathrm{q})}{1 - Q(\mathrm{q})[1 - L(\mathrm{q})P_\Delta(\mathrm{q})]} \right| < 1,\ \ \forall P_\Delta(\mathrm{q}), \forall \mathrm{q} \in \mathcal{D}, \tag{10}$$

where W_u is the input weight function. It is worth noting that (5) concerns monotonic convergence of input signal in the trial domain thus (10) is also sufficient for all input signals in the trial domain.

2.5. Multi-objective ILC problem

In this work, the above considered ILC objectives are combined together in one constrained optimization problem $Q(\mathrm{q})$ and $L(\mathrm{q})$:

$$\begin{array}{ll} \underset{Q(\mathrm{q}),\, L(\mathrm{q})}{\text{minimize}} & \text{convergence speed} \\ \text{subject to} & \text{robust convergence (5)} \\ & \text{robust performance (8)} \\ & \text{input constraint (10).} \end{array} \tag{11}$$

The main idea is to optimize the convergence speed considering the given tracking performance specification, and taking into account robustness and input constraint. The solution is elaborated in the next section, where we will show later that it can be reformulated as a convex problem.

3. Main Results

This section first reevaluates the ILC design objectives and then solve the proposed optimization problem (11).

3.1. ILC objectives

First, we consider the robust convergence (RC) condition (5). It is easily seen that

$$\text{(RC)} \Leftrightarrow \sup_{\Delta \in \mathcal{B}_\Delta} |Q(1 - L\hat{P}) - QL\hat{P}W\Delta| < 1, \ \forall \omega \tag{12}$$

$$\Leftrightarrow |Q(1 - L\hat{P})| + |QL\hat{P}W| < 1, \quad \forall \omega \tag{13}$$

$$\Leftrightarrow |QL\hat{P}W| < 1 - |Q - QL\hat{P}|, \quad \forall \omega, \tag{14}$$

where the argument q is abbreviated for convenience.

Lemma 3.1 (Necessary Condition)**.** *Consider the uncertain system (2), if the ILC robust convergence condition*

$$|Q(1 - LP_\Delta)| < 1, \quad \forall P_\Delta, \forall \omega, \tag{15}$$

is guaranteed, then the following inequality is satisfied

$$\left| \frac{QL\hat{P}W}{1 - Q + QL\hat{P}} \right| < 1, \quad \forall \omega \tag{16}$$

Proof. *From the following triangle inequality*

$$|1 - (Q - QL\hat{P}) + (Q - QL\hat{P})| \leq |1 - (Q - QL\hat{P})| + |(Q - QL\hat{P})|, \quad \forall \omega, \tag{17}$$

hence

$$1 - |Q - QL\hat{P}| \leq |1 - Q + QL\hat{P}|, \quad \forall \omega, \tag{18}$$

then from (14) and (18), yielding

$$|QL\hat{P}W| < |1 - Q + QL\hat{P}|, \quad \forall \omega. \tag{19}$$

This inequality leads to (16).

Next, robust performance (8) is investigated, as shown in the following lemma.

Lemma 3.2. *Consider the uncertain system (2), if the ILC robust convergence condition (12) is satisfied then the ILC robust performance condition*

$$\left| W_{\mathrm{p}} \frac{1-Q}{1-Q(1-LP_{\Delta})} \right| < 1, \ \ \forall P_{\Delta}, \forall \omega \tag{20}$$

is equivalent to

$$\left| \frac{W_{\mathrm{p}}(1-Q)}{1-Q+QL\hat{P}} \right| + \left| \frac{QL\hat{P}W}{1-Q+QL\hat{P}} \right| < 1, \ \ \forall \omega. \tag{21}$$

Proof. *Robust performance (RP) is satisfied if the worst-case function with respect to* $\forall \Delta \in \mathcal{B}_{\Delta}$ *is less than 1, hence*

$$(\mathrm{RP}) \ \Leftrightarrow \ \sup_{\Delta \in \mathcal{B}_{\Delta}} \left| \frac{W_{\mathrm{p}}(1-Q)}{1-Q+QL\hat{P}+QL\hat{P}W\Delta} \right| < 1, \ \forall \omega. \tag{22}$$

The supremum is actually a maximum and is achieved when Δ *is selected at each frequency such that* $|\Delta| = 1$ *and the terms* $(1-Q+QL\hat{P})$ *and* $QL\hat{P}W\Delta$ *(which are complex numbers) point in opposite directions. We get*

$$\sup_{\Delta \in \mathcal{B}_{\Delta}} \left| \frac{W_{\mathrm{p}}(1-Q)}{1-Q+QL\hat{P}+QL\hat{P}W\Delta} \right| = \frac{|W_{\mathrm{p}}(1-Q)|}{|1-Q+QL\hat{P}| - |QL\hat{P}W|}. \tag{23}$$

Note that from the necessary condition of ILC robust convergence (16), it is guaranteed that $|1-Q+QL\hat{P}| - |QL\hat{P}W| > 0$*. By substituting (23) to (22), the robust performance condition (21) is derived.*

Satisfying robust performance guarantees nominal performance but does not necessarily guarantees robust convergence since (16) is a necessary condition. It is worth stressing that from (16) and (21), robust performance is automatically obtained within a factor of 2 when both nominal performance and robust convergence are satisfied. Hence, it is justified to relax the robust performance constraint in the optimization problem (11) by using both nominal performance and robust convergence constraints. Furthermore, it is observed that:

$$\frac{1-Q}{1-Q+QL\hat{P}} + \frac{QL\hat{P}}{1-Q+QL\hat{P}} = 1, \ \ \forall \omega. \tag{24}$$

In other words, tracking performance is limited due to model uncertainty, and W_{p} plays an important role in analyzing the design trade-off between tracking performance and convergence speed. The design of W_{p} is intuitive for control engineers since the idea is analogous to the sensitivity weight function in feedback control ([19]). The difference is that for ILC design W_{p} is not restricted to causal functions.

Finally, the input constraint objective is given.

Lemma 3.3. *Consider the uncertain system (2), the ILC input constraint condition*

$$\left| W_{\mathrm{u}} \frac{QL}{1-Q(1-LP_{\Delta})} \right| < 1, \ \ \forall P_{\Delta}, \forall \omega \tag{25}$$

is given by

$$\left| \frac{W_{\mathrm{u}}QL}{1-Q+QL\hat{P}} \right| + \left| \frac{QL\hat{P}W}{1-Q+QL\hat{P}} \right| < 1, \ \ \forall \omega. \tag{26}$$

Proof. *The proof is given similarly to the robust performance case in Lemma 3.2.*

3.2. ILC design

From the results of the previous subsection, we relax the original optimization problem (11) by considering the following optimization problem:

$$\underset{Q,\,L}{\text{minimize}} \quad |Q(1-L\hat{P})|, \quad \forall\omega \tag{27a}$$

$$\text{subject to} \quad \left|Q(1-L\hat{P})\right| + \left|QL\hat{P}W\right| < 1, \quad \forall\omega \tag{27b}$$

$$\left|\frac{W_{\mathrm{p}}(1-Q)}{1-Q+QL\hat{P}}\right| < 1, \quad \forall\omega \tag{27c}$$

$$\left|\frac{W_{\mathrm{u}}QL}{1-Q+QL\hat{P}}\right| < 1, \quad \forall\omega. \tag{27d}$$

This problem is not convex and hard to solve. However, it is interesting to note that from (18):

$$1-|Q-QL\hat{P}| \leq |1-Q+QL\hat{P}|, \quad \forall\omega, \tag{28}$$

this yields

$$|Q-QL\hat{P}| \leq \gamma \;\;\Rightarrow\;\; 1-\gamma \leq |1-Q+QL\hat{P}|, \;\; \forall\omega. \tag{29}$$

Therefore,

$$|QL\hat{P}W| < 1-\gamma \Rightarrow |Q-QL\hat{P}| + |QL\hat{P}W| < 1, \forall\omega, \tag{30}$$

$$|W_{\mathrm{p}}(1-Q)| < 1-\gamma \;\Rightarrow\; \left|\frac{W_{\mathrm{p}}(1-Q)}{1-Q+QL\hat{P}}\right| < 1, \;\; \forall\omega, \tag{31}$$

$$|W_{\mathrm{u}}QL| < 1-\gamma \;\Rightarrow\; \left|\frac{W_{\mathrm{u}}QL}{1-Q+QL\hat{P}}\right| < 1, \;\; \forall\omega. \tag{32}$$

As a result, the following optimization problem is proposed to compute the learning controller:

$$\underset{Q,\,L}{\text{minimize}} \quad \gamma \tag{33a}$$

$$\text{subject to} \quad 0 \leq \gamma < 1 \tag{33b}$$

$$|Q(1-L\hat{P})| \leq \gamma, \quad \forall\omega \tag{33c}$$

$$|QL\hat{P}W| < 1-\gamma, \quad \forall\omega \tag{33d}$$

$$|W_{\mathrm{p}}(1-Q)| < 1-\gamma, \quad \forall\omega \tag{33e}$$

$$|W_{\mathrm{u}}QL| < 1-\gamma, \quad \forall\omega. \tag{33f}$$

Using the change of variable: $\hat{L} = QL$, then the constraints are linear functions of Q and $\hat{L}$. Next, we consider Q and $\hat{L}$ as finite impulse response (FIR) learning filters, consisting of both causal and noncausal taps, given by

$$\begin{aligned} Q(\mathrm{q}) &= q_{-n}\mathrm{q}^{-n} + \ldots + q_{-1}\mathrm{q}^{-1} + q_0 + q_1\mathrm{q}^1 + \ldots + q_{\bar{n}}\mathrm{q}^{\bar{n}}, \\ \hat{L}(\mathrm{q}) &= \hat{l}_{-m}\mathrm{q}^{-m} + \ldots + \hat{l}_{-1}\mathrm{q}^{-1} + \hat{l}_0 + \hat{l}_1\mathrm{q}^1 + \ldots + \hat{l}_{\bar{m}}\mathrm{q}^{\bar{m}}. \end{aligned} \tag{34}$$

Consequently, the constraints of (33) are linear functions of the impulse response parameters. Notice that noncausal learning filters are essential to obtain high tracking performance and deal with non-minimum phase systems model [8].

The resulting optimization problem is semi-infinite since the constraints require evaluation for infinitely values of ω. Semi-infinite constraints are handled by choosing a reasonable finite set of frequency samples $0 < \omega_1 < \ldots < \omega_M$ (frequency gridding) up to the Nyquist frequency. Then we can replace the semi-infinite constraints with the finite set of constraints at each of the given frequencies. For example, the constraint $|Q(1 - L\hat{P})| \leq \gamma, \forall\omega$ is replaced by $|Q(j\omega_k)(1 - L(j\omega_k)P(\hat{j\omega_k}))| \leq \gamma,\ k = 1, \ldots, M$. Since (33) is a linear program, the computational cost of the optimization problem is linearly proportional to M. We can choose a large enough M such that this frequency gridding is no practical issue. A standard rule of thumb is to choose $M \approx 15h$, where h is the total order of the filters, as discussed in [20]. In addition, parametric models are not required in the ILC design (33), since only the systems frequency response function data at the given finite set of frequencies is need.

3.3. Extensions

In this subsection, we present two extensions of the proposed ILC design. The first extension concerns formulation of the ILC optimization problem with different objectives and constraints. And the second extension considers a different class of learning filters, that is, infinite impulse response filters.

3.3.1. Different objectives and constraints

In (11), we optimize the convergence speed for given desired tracking performance and robustness, input constraints. Another idea is to optimize the tracking performance considering a specified convergence speed and other constraints. In other words, the problem is represented by

$$\begin{array}{ll} \underset{Q(\mathrm{q}),\, L(\mathrm{q})}{\text{minimize}} & \text{tracking performance} \\ \text{subject to} & \text{robust convergence} \\ & \text{convergence speed constraint} \\ & \text{input constraint.} \end{array} \tag{35}$$

For this case, the convergence speed condition is defined as

$$|W_{\mathrm{c}}(Q - QL\hat{P})| \leq 1, \quad \forall\omega. \tag{36}$$

where W_{c} is the convergence speed weight selected by the designer.

The optimization problem is reformulated as a convex problem similarly as the proposed approach (33). Following the same steps as in section III.B, (35) transforms into following convex problem:

$$\begin{array}{ll} \underset{Q,\, L}{\text{minimize}} & \gamma \\ \text{subject to} & |W_{\mathrm{c}}Q(1 - L\hat{P})| \leq 1,\ \ \forall\omega \\ & |W_{\mathrm{c}}QL\hat{P}W| < |W_{\mathrm{c}}| - 1,\ \ \forall\omega \\ & |W_{\mathrm{p}}(1 - Q)| < \gamma(1 - |W_{\mathrm{c}}^{-1}|),\ \ \forall\omega \\ & |W_{\mathrm{u}}QL| < \gamma(1 - |W_{\mathrm{c}}^{-1}|),\ \ \forall\omega. \end{array} \tag{37}$$

3.3.2. Infinite impulse response (IIR) filters

The Q-filter and learning function are not necessarily limited to FIR filters. We can also extend the results to IIR filters, for example, linearly parameterized controllers, as given by

$$\begin{aligned} Q(\mathrm{q}) &= q_0\phi_0(\mathrm{q}) + q_1\phi_1(\mathrm{q}) + \ldots + q_n\phi_n(\mathrm{q}), \\ \hat{L}(\mathrm{q}) &= \hat{l}_0\varphi_0(\mathrm{q}) + \hat{l}_1\varphi_1(\mathrm{q}) + \ldots + \hat{l}_m\varphi_m(\mathrm{q}), \end{aligned}$$

where $q_0, q_1, \ldots, l_0, l_1, \ldots$ are the filter parameters, and $\phi_i(\mathrm{q})$, $\varphi_i(\mathrm{q})$ are a selected set of orthonormal basis functions [21]. This technique requires the selection of the basis functions, and can be useful in cases where the required length of the FIR filters is too large.

4. Numerical Illustration

This section illustrates the potential of the proposed ILC design approach. The numerical validation comprises two parts. First, we compare the proposed ILC approach with a conventional model-inversion ILC design. Second, we analyze the design trade-off between converged tracking error and convergence speed by considering different performance weights $W_{\mathrm{p}}(\mathrm{q})$ and multiplicative uncertainty weights $W(\mathrm{q})$ in our method.

The considered system is a non-minimum phase system:

$$\hat{P}(s) = \frac{-1000(s-\omega_3)}{(s/\omega_1+1)[(s/\omega_2)^2+2\zeta(s/\omega_2)+1]}; \tag{38}$$

where $\omega_1 = 2\pi$, $\omega_2 = 100\pi$, $\omega_3 = 20\pi$, and $\zeta = 0.6$. The model is discretized with a sampling time $T = 0.001$s. Fig. 1 shows the nominal model. The gray area represents the considered system uncertainty corresponding to the uncertainty weight $W(\mathrm{q})$ shown in figure 2. The performance weight $W_{\mathrm{p}}(\mathrm{q})$ is designed such that the converged tracking error is small at low frequencies up to about 60Hz. The input weight $W_{\mathrm{u}}(\mathrm{q})$ is used to constraint the input signal at high frequencies ($f \geq 200$Hz). These weights are also shown in Fig. 2.

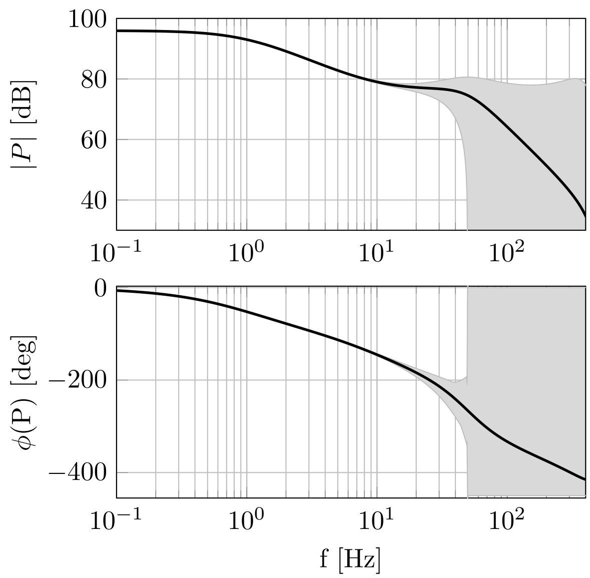

Figure 1: Bode-diagrams of uncertain plants

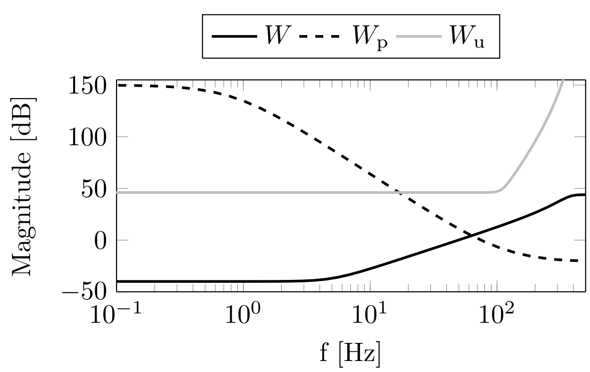

Figure 2: Uncertainty weight and performance weight

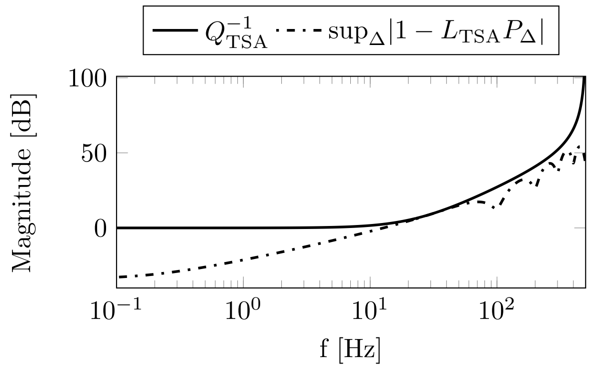

Figure 3: Model-inversion TSA-based ILC design

4.1. Comparison with model-inversion ILC

When applying the model-inversion ILC approach, $L(\mathrm{q})$ is a stable approximation of the inverse of the non-minimum phase system model. Several model inversion approximation schemes exist [22]. In this paper we apply the truncated series approximation (TSA) scheme with 10 noncausal filter tabs [23]. $Q(\mathrm{q})$ is initially designed as a low-pass Butterworth filter of order 1. This filter is then applied forward and backward to the signal, yielding a 2nd-order zero-phase low-pass filter. The cut-off frequency of the Butterworth filter is chosen such that the resulted zero-phase filter satisfies the robust convergence condition (5), as given in Fig. 3.

In our multi-objective ILC approach, $Q(\mathrm{q})$ and $L(\mathrm{q})$ are solutions of the optimization problem (33). $L(\mathrm{q})$

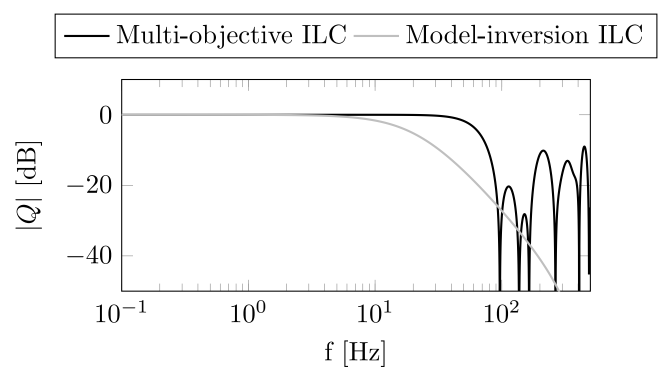

Figure 4: Designed Q-filters

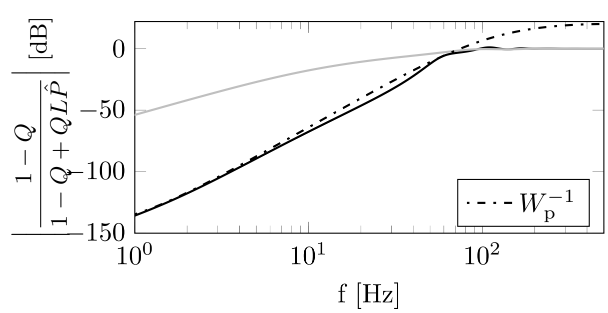

Figure 5: Tracking performance

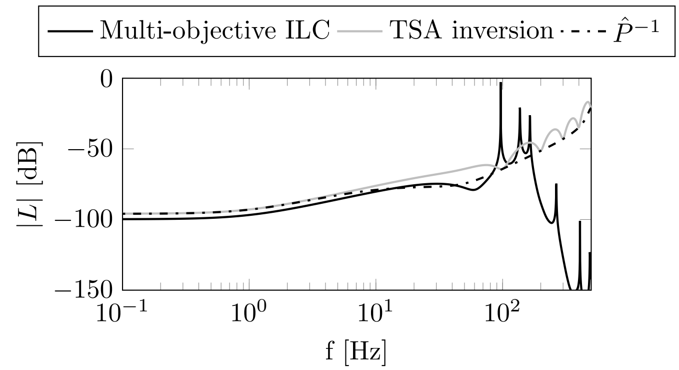

Figure 6: Designed learning function $L(\mathrm{q})$

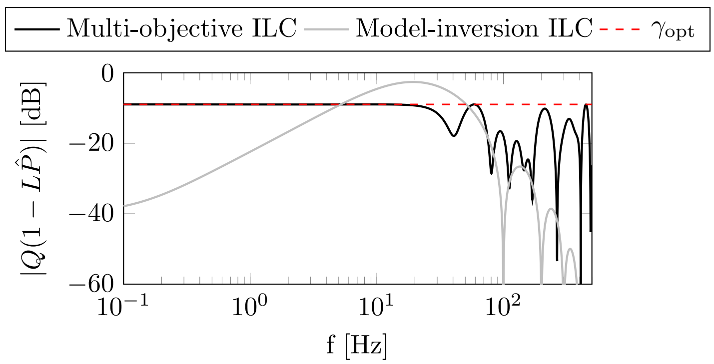

Figure 7: Robustness and convergence speed

has the same number of noncausal filter tabs as the learning function in the model-inversion ILC. $Q(\mathrm{q})$ is a zero-phase FIR filter (34), where the impulse responses are symmetric about q_0 and the order $\bar{n} = 15$. The number of frequency samples is 5000 samples. The Matlab-based convex modeling framework CVX with the solver MOSEK to is used to solve the optimization problem (33) [24]. It takes 14 seconds to compute the solution in our standard laptop with an Intel Core I5 processor.

Fig. 4 and Fig. 5 show the Q-filter and tracking performance function, respectively. Clearly, the proposed multi-objective ILC design approach achieves significantly higher bandwidth than the model-inversion ILC design. This demonstrates the main advantage of our one-step optimization based ILC approach. In Fig. 5, the performance condition (27c) is also verified, indicating the capacity of shaping the ILC tracking performance function of the multi-objective approach.

Fig. 6 and Fig. 7 show the learning function $L(\mathrm{q})$ and the convergence speed, respectively, of both approaches. The optimal solution γ_{opt} of the optimization problem (33) is also illustrated. From Fig. 7, it can be seen that the convergence speed of our multi-objective approach is frequency independent up to about 20Hz, and is at least γ_{opt} for any frequency. The model-inversion ILC converges faster below 5Hz and above 50Hz, at the price of slower convergence between 5Hz and 50Hz. This faster convergence below 5Hz is realized by the model-inversion approach by a good match of $L(\mathrm{q})$ with the inverse of the system, and above 50Hz is realized by the roll-off the Q-filter. The learning function $L(\mathrm{q})$ of the multi-objective approach matches the inverse of the system model less accurately at low frequencies, leaving more freedom to approximate the inverse model better in the frequency range 5 - 50Hz in order to realize a more uniform convergence speed over a wider frequency range. Fig. 6 also shows that our learning function is extremely small at high frequencies $f \geq 200$Hz. This is because the input weight function was imposed such that learning at these frequencies is not important. Fig. 8 shows the input performance of both approaches.

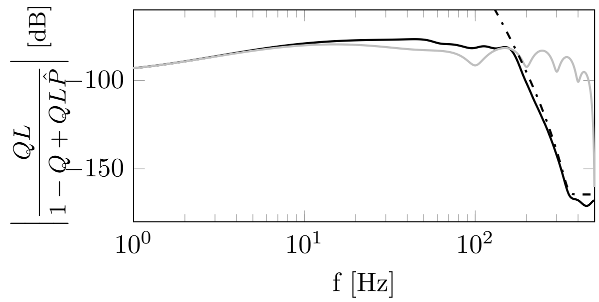

Figure 8: Input performance: multi-objective ILC (solid), model-inversion ILC (gray), and input weight inversion W_u^{-1} (dash)

Clearly, the input gain at high frequencies of our multi-objective approach is substantially smaller than the model-inversion ILC, which is usually more desirable.

Notice that with the given performance weight $W_\mathrm{p}(\mathrm{q})$, the optimal convergence speed γ_opt also depends on the input constraint condition and the assigned number of filter tabs of $Q(\mathrm{q})$ and $L(\mathrm{q})$ in the optimization problem (33). In order to further investigate these dependencies, we apply the proposed ILC design for two other cases. First, the input constraint is ignored in the optimization problem (33). Second, the number of tabs of $Q(\mathrm{q})$ is increased, i.e. $\bar{n} = 30$ (the order of $L(\mathrm{q})$ is remained). The results are given in Table I. Clearly, input constraint will lower the convergence speed. In addition, the higher Q-filter order yields higher convergence speed.

Table 1: Optimal convergence speed γ_opt

	Input constraint	Without input constraint
$\bar{n} = 15$	0.36	0.27
$\bar{n} = 30$	0.11	0.09

The above analyses are for all uncertain models in the uncertainty set (2). Next, we apply both ILC approaches to the one particular uncertain system of the considered set in order to validate the results in the time domain, that is, with

$$\Delta(s) = \frac{s-1}{s+1}. \tag{39}$$

The reference signal is a smoothed step trajectory. The results are shown in Fig. 9 and Fig. 10. It can

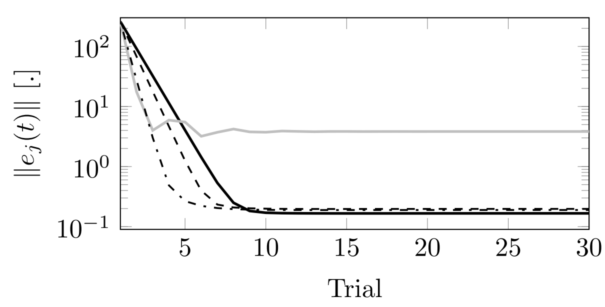

Figure 9: Multi-objective ILC designs: $\bar{n} = 15$ and with input constraint (solid), $\bar{n} = 15$ and without input constraint (dash), $\bar{n} = 30$ and with input constraint (dash-dot), and comparison with model-inversion ILC (gray)

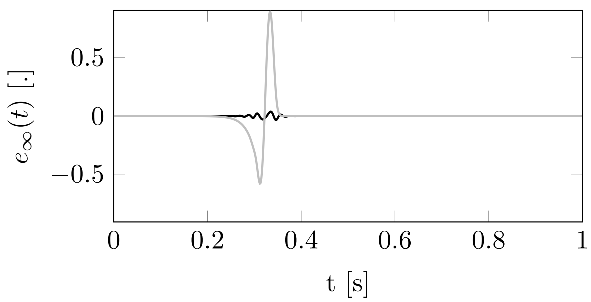

Figure 10: Asymptotic tracking error: multi-objective ILC (black) and model-inversion ILC (gray)

be seen that both approaches achieve convergence. The convergence speed with $n = 15$ (solid line) of our multi-objective approach is considerably lower than the model-inversion approach (grey line), however the asymptotic tracking error is considerably smaller. Without sacrificing on the asymptotic tracking error, the convergence speed of the multi-objective approach considerably increases when the input constraints are removed and the Q-filter order $\bar{n}$ is increased from 15 to 30. In the last case a similar convergence speed as the model-inversion approach is obtained.

4.2. Class of uncertain systems

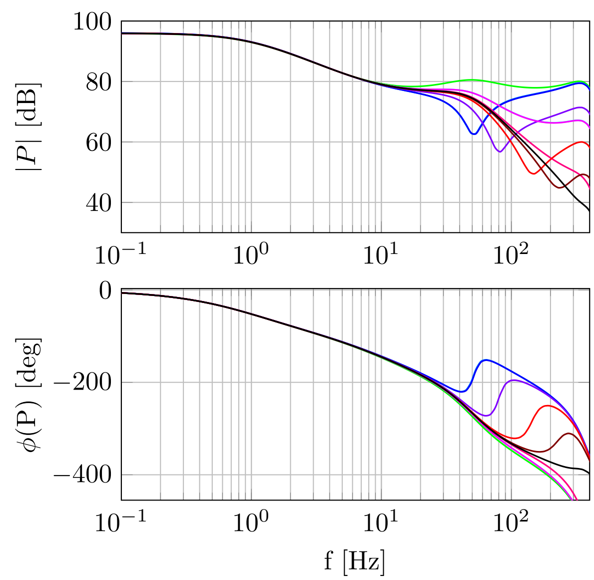

Figure 11: Bode-diagrams of the ten random uncertain plants

In this subsection, we validate the proposed approach with different uncertain systems in the considered uncertainty set (2). First, random LTI uncertainty models $\Delta(\mathrm{q})$ where $\|\Delta(\mathrm{q})\|_\infty \leq 1$ are generated using the MATLAB command *ultidyn*, yielding ten different system plants $P_\Delta(\mathrm{q})$. The uncertain plants are shown in Fig. 11. Next, we apply the designed controller with the case $\bar{n} = 15$ to these plants. Fig. 12 shows that our algorithm achieves monotonic convergence and high tracking performance for all considered systems. Finally, we simulate again with 1000 generated random uncertainty models $\Delta(\mathrm{q})$. The controller is applied to these uncertain plants and then the tracking performance is averaged. The result is also shown in the in Fig. 12. This figure confirms the effectiveness of the proposed design for a large set of uncertain plants in the considered uncertainty set.

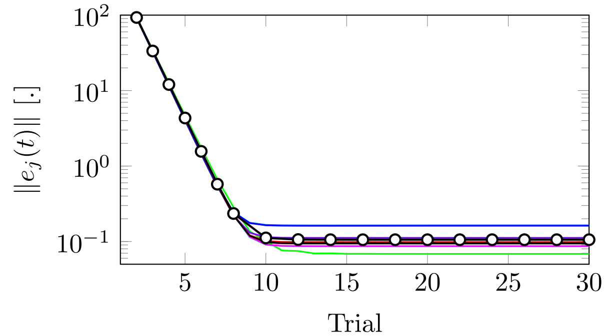

Figure 12: Tracking performance with the ten considered uncertain plants and the average tracking performance with 1000 random uncertain plants (circle markers line)

4.3. Design trade-off analysis

In this subsection, the trade-off between tracking performance and convergence speed of our multi-objective ILC approach is further analyzed. The convexity of the obtained optimization problem allows a straightforward and efficient computation of the trade-off curves. The results are obtained by scaling the performance weight $W_{\mathrm{p}}(\mathrm{q})$ and the multiplicative uncertainty weight $W_{\mathrm{p}}(\mathrm{q})$ in the optimization problem (33), i.e. replace $W_{\mathrm{p}}(\mathrm{q})$ by $a \times W_{\mathrm{p}}(\mathrm{q})$ with $a = 0.2, 0.4, \ldots, 1.4$, and replace $W(\mathrm{q})$ by $b \times W(\mathrm{q})$ with $b = 1.1, 1.2, \ldots, 1.6$.

In Fig. 13, we show the trade-off curves between the asymptotic tracking error $\|e_\infty(t)\|$ and the optimal convergence speed γ_{opt} considering different performance weights and uncertainty weights. The asymptotic tracking errors are calculated using the uncertain system (39). Clearly, the results demonstrate that the larger performance weight results in lower convergence speed but smaller converged tracking error. In addition, the larger uncertainty weight yields lower convergence speed and larger converged tracking error. These conclusions are further confirmed in Fig. 14 and Fig. 15, where tracking errors in the trial domain are plotted with different performance weights and uncertainty weights, respectively.

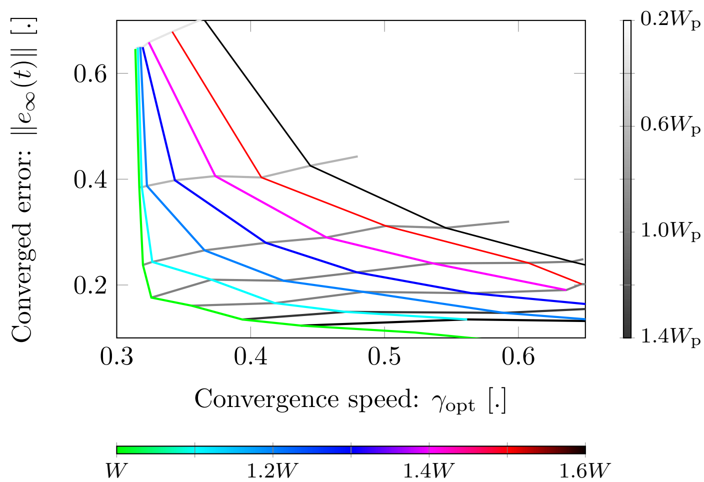

Figure 13: Trade-off curves: convergence speed vs asymptotic tracking error with different performance weights and multiplicative uncertainty weights

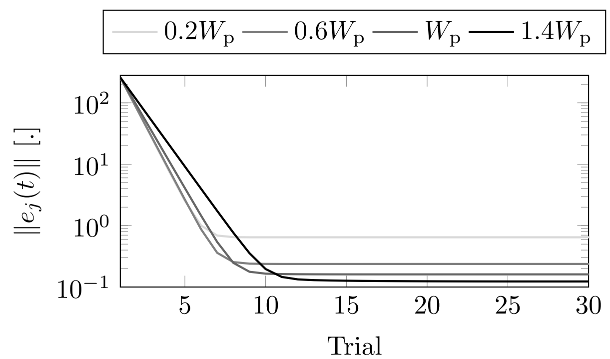

Figure 14: Tracking performance with uncertainty weight $W(\mathrm{q})$ and different performance weights

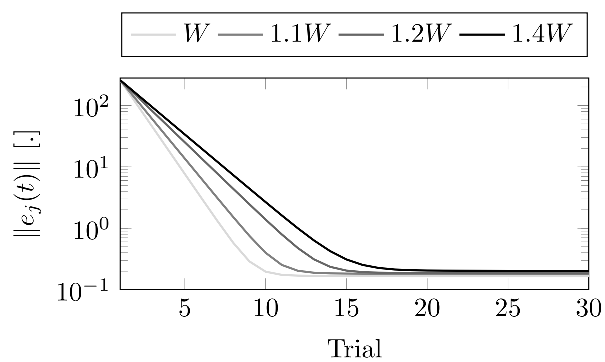

Figure 15: Tracking performance with performance weight $W_\mathrm{p}(\mathrm{q})$ and different multiplicative uncertainty weights

5. Conclusion

The main contribution of this paper is an ILC design that can incorporate multiple ILC design objectives: robust convergence accounting for model uncertainty, tracking performance, convergence speed, and input constraint. The proposed design approach corresponds to a convex optimization problem that can be solved efficiently for optimal learning parameters $Q(\mathrm{q})$ and $L(\mathrm{q})$ simultaneously. This design also allows the computation of the trade-off curves between objectives. Detailed analysis of the proposed optimal ILC approach and comparison with the conventional model-inversion ILC are presented for a uncertain non-minimum phase plant.

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